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Effective Diffusion Homogenization of Cross Sections for Pressurized Water Reactor Core Calculations

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Abstract—*A consistent, effective diffusion homogenization method for cross sections for pressurized water reactors (PWRs) is presented. It can be applied to obtain cell-averaged as well as assembly-averaged cross sections. Since no additional parameters are necessary, standard diffusion codes can be used. It is shown that a few-group diffusion calculation over a fuel assembly compares favorably with a transport calculation. This allows transport theory methods to be restricted to lattice cell calculations. The application of the critical albedo search in fuel assembly calculations provides for the approximate treatment of the radial leakage and helps to decouple calculations over individual fuel assemblies from their surroundings. Examples of calculations on unit cells, fuel assemblies, and realistic PWR core geometry are presented.*

I. INTRODUCTION

Homogenization of heterogeneous regions in reactor calculations is necessary because in routine applications, detailed calculations are restricted to relatively small heterogeneous regions because of computer limitations, in spite of the increased capacity of modern computers. The simplest homogenization method uses flux and volume weighting of the cross sections. It is referred to as the FVH method in this paper. The method is attractive because of its simplicity, but it suffers severe drawbacks due to its inaccuracy, particularly when relatively large, heterogeneous regions containing strong absorbers are to be homogenized.

The problem of cross-section homogenization has been addressed by several authors, and satisfactory results have been obtained in their particular areas of application. For homogenization of fuel assembly cross sections, for example, Henry, Worley, and Morshed[1] derived effective diffusion parameters by a response matrix method. In the equivalence theory (ET), Koebke[2] defined direction-dependent diffusion constants and flux heterogeneity factors. The generalized equivalence theory (GET) by Smith[3] introduced flux discontinuity factors and followed essentially the same approach: With additional parameters and with the use of modified diffusion codes, the reaction rates of reference heterogeneous problems in equivalent homogenized solutions were preserved exactly. A further step in the development of the homogenization methods was taken by Koebke[4] in the simplified equivalence theory (SET), which was tested for pressurized water reactor (PWR) applications. With the definition of a single additional parameter per group and the use of standard diffusion codes, satisfactory results were obtained.

For lattice cell homogenization, the Superhomogenéisation (SPH) method was described by Kavenoky[5] and recently elaborated by Hébert and Benoist.[6,7] It is based on collision probability theory to produce equivalent homogenized lattice cell transport cross sections for whole-assembly transport calculations. A variant of the method has also been presented by Hébert[7] to derive diffusion equivalent cell cross sections, primarily intended for two-group pin-by-pin global reactor calculations. In this work, the radial leakage is not treated explicitly.

These methods rely on transport calculations for the pin cell and for the whole assembly and its surroundings. When the ET and GET methods are applied, a modified diffusion code for global reactor calculations is also required.

Further work on cross-section homogenization methods with the use of discontinuity factors has been reported, such as that presented by Zhang, Rizwan-uddin, and Dorning.[9,10] Their method has been demonstrated on several one-group cases, one of them

representing a fuel assembly problem with realistic geometry. The rigorous mathematical derivation offers the additional advantage of producing local power distributions without further approximations, but the method is rather elaborate even in the one-group case. It may find its place in the next generation of reactor core calculation software. On the other hand, our approach relies more on simple proven methods, with extensions and simplifications so as to introduce minimal additional errors. The proposed method is compatible with existing lattice and diffusion codes.

The aim of this paper is to describe an effective diffusion homogenization (EDH) method for cross-section homogenization. It is derived by demanding conservation of reaction rates and surface partial currents in an average sense. As is shown later, for fuel assembly homogenization, the EDH method is a special case of the SET method (or the GET method) where the same discontinuity factor is assumed on all faces of the homogenized region and used to normalize all cross sections, including the diffusion constant. The main differences in the EDH method are a further simplification compared with the SET method and its application to derive effective homogenized diffusion parameters for lattice cells as well as for whole assemblies. This allows fuel assembly calculations to be performed in the multigroup diffusion approximation without any significant loss of accuracy. Reflector diffusion parameters are derived in a manner equivalent to the SET method.

In the EDH method, the radial leakage is treated explicitly by the appropriate selection of the boundary conditions. For cell calculations, these are obtained from the heterogeneous transport solution. In the heterogeneous assembly calculation, the radial leakage is treated by imposing *critical albedo* boundary conditions. The same boundary conditions are used in the equivalent homogenized problems. The critical albedo assumption provides that the radial leakage from a fuel assembly is correct *on average* in a critical core (which is usually of interest) and helps to decouple individual fuel assembly calculations from their surroundings. The advantage of the EDH method over the diffusion variant of Hébert's SPH method[7] is the simple, explicit treatment of the radial leakage and the application of the method in few-group rather than two-group form. The analogous "surface leakage model" appears as an option in the work by Hébert and Benoist[6] and Hébert and Mathonniére[8] but only for fuel assembly homogenization, in a more elaborate way.

The main characteristics of the proposed method are as follows:

1. A standard diffusion code is used for fuel assembly as well as for global reactor calculations. Transport methods are limited to lattice cell calculations.

2. No additional parameters are introduced in homogenized cross sections.

3. Radial leakage is treated explicitly. For fuel assembly calculations, this is done by a critical albedo search, thus approximately treating the surrounding medium of a fuel assembly.

4. Because of the approximate treatment of the fuel assembly surroundings, the assembly cross sections are environment independent (i.e., the cross sections of two identical assemblies placed at different locations in the core are the same).

The practical applications presented in this paper indicate that the proposed method is suitable for PWR-type calculations. It introduces negligible error even for regions containing strong absorbers and offers significant savings in computation time with respect to other homogenization methods, particularly transport-solution-based ones.

II. DESCRIPTION OF THE EFFECTIVE DIFFUSION HOMOGENIZATION METHOD

II.A. Basic Principles of the Method

Consider a heterogeneous region embedded in some other (heterogeneous) medium. Define an equivalent homogeneous region, which has the same volume and shape as the heterogeneous region of interest. A set of nuclear parameters is required, so that the homogenized region will respond as much as possible in the same way as the heterogeneous region.

Let us assume that a reference solution (i.e., neutron flux distribution) for the heterogeneous region to be homogenized is available (either by an explicit solution over the region of interest and its surroundings or by an appropriate selection of boundary conditions to treat the surroundings approximately). Either transport or diffusion theory calculations can be applied.

In defining equivalent homogenized cross sections for a certain region, it is customary to impose conditions on the conservation of the group average flux and the reaction rates. When the effective neutron multiplication factor k_{eff} and the group average flux values in the reference heterogeneous region match those in the equivalent homogenized region and reaction rates are conserved, the total leakage per group, i.e., the net current integrated over the outer boundary of the region, is also conserved. This follows from the neutron balance condition. With these constraints, partial currents on the region boundary in general cannot be conserved without introducing additional parameters. Therefore, for a given partial incoming current, the homogenized region (assuming FVH cross sections) will respond with incorrect outgoing currents, compared with the reference heterogeneous case.

The information on the neutron flux is not really needed in the solution, except to calculate the reaction rates, fission density in particular. If reaction rate conservation is imposed *a priori*, the condition on average

flux can be relaxed, and the condition on partial currents conservation can be imposed instead.

In deriving the EDH method, definitions of the effective diffusion cross sections and the effective flux similar to those in the SPH method are applied:

$$\tilde{\Sigma}_{xg} = \mu_g \Sigma_{x,g} \quad \text{and} \quad \tilde{\phi}_g = \frac{1}{\mu_g} \phi_g \ , \tag{1}$$

where

$\Sigma_{x,g}$ = FVH average cross section for reaction x in group g

ϕ_g = average flux

$\tilde{\Sigma}_{x,g}, \tilde{\phi}_g$ = corresponding effective cross section and flux.

The diffusion constant is treated in the same way as the cross sections, except in the case of the reflector constants, as is shown later. Scaling parameter μ_g is defined such that the partial currents on the boundary are conserved on average. This is an additional assumption, compared with some other methods, which is justified when the partial currents are approximately constant along the region boundary.

To calculate the partial currents on the boundary of the homogenized region, it is convenient to derive an analytical solution of the problem. This is possible because the homogenized regions usually have simple geometry. Exact solutions can be derived for circular and slab regions, while for rectangular regions, the separability assumption is required. Although this assumption is not entirely justified, it was found to have a negligible effect on the final results in view of other approximations.

A set of equations could be constructed to obtain the μ_g parameters in closed form; however, because of its simplicity and negligible calculational effort, an iterative procedure is preferred. Iterations consist of the following steps:

1. Define an equivalent homogeneous region of the same volume and shape for which a reference heterogeneous solution is available.

2. Define an initial guess for the cross sections and the diffusion constant (e.g., by the FVH method).

3. Solve the diffusion equation for the homogeneous region using boundary conditions on the boundary flux values from the reference solution. Note that the volume average flux and the average boundary net current will differ from the reference solution.

4. Define the scaling factor μ_g as the ratio of the initial and the calculated volume average flux in group g.

5. Scale the cross sections (including the diffusion constant) according to Eq. (1) to conserve the reaction rates.

6. Reiterate beginning from item 3 until the difference between the reference and the calculated leakage is negligible compared with the total neutron source in each group.

When EDH cross sections are used in the usual diffusion calculations over homogenized regions, the solution for the neutron flux is such that the reaction rates are conserved. Note that the true average flux in a homogenized region can also be reconstructed if the scaling parameter μ_g is saved together with the other cross sections for each homogenized region. The EDH correction to the cross sections does not affect the infinite-medium multiplication factor k_∞ since the scaling factors μ_g cancel out.

This procedure is adequate and well suited for regions in which partial currents along the boundary do not change significantly. The reflector is a specific case where this condition is severely violated even in one-dimensional geometry. Furthermore, in the reflector, it is strictly necessary to conserve the partial currents on the internal boundary, while the conditions on the outer boundary are of less significance. This request can be accommodated in the proposed procedure quite simply by imposing the zero-flux condition on the external boundary. To match the partial currents on the internal boundary, another iteration loop is added in which the group diffusion constants are varied until full consistency of partial currents on the internal boundary between the reference and the analytic solution over the homogenized reflector is obtained. In this specific case, the method is applied in a one-dimensional form and is practically equivalent to the SET method.

The steps in deriving equivalent diffusion parameters by the EDH method are quite trivial, except step 3, which requires the analytic solution of the diffusion equation in the homogenized region. This is described in Sec. II.B for some typical shapes of homogenized regions.

II.B. Solution of the Diffusion Equation

Consider the multigroup neutron diffusion equation in one-dimensional geometry for a homogeneous slab or a circular region:

$$-D_{(g)} \nabla^2 \phi_{(g)} + \Sigma_{r(g)} \phi_{(g)} = \frac{\chi_{(g)}}{k_{eff}} \sum_h \nu_{(h)} \Sigma_{f(h)} \phi_{(h)} + \sum_{h \neq g} \Sigma_{s(h \to g)} \phi_{(h)} \ , \tag{2}$$

where

$$\Sigma_{r(g)} = \Sigma_{a(g)} + D_{(g)} B^2 + \sum_{h \neq g} \Sigma_{s(g \to h)}$$

$D_{(g)}$ = group g diffusion constant

$\Sigma_{a(g)}$ = group g absorption cross section

$\Sigma_{f(g)}$ = group g fission cross section

$\nu_{(g)}$ = number of neutrons per fission in group g

$\chi_{(g)}$ = fraction of fission neutrons born in group g

$\phi_{(g)}$ = neutron flux in group g

B^2 = transverse buckling.

Divide Eq. (2) for each group by $D_{(g)}$. Define the Laplace operator:

$$\nabla^2\xi - \lambda^2\xi = 0 \ . \tag{3}$$

Use the eigenfunctions ξ of this operator as trial functions. On substitution and rearrangement, Eq. (2) can be written in matrix form:

$$(\mathbf{G} - \lambda^2\mathbf{I})\boldsymbol{\Xi} = 0 \ , \tag{4}$$

where

$\boldsymbol{\Xi}$ = vector of space-dependent functions with elements $\alpha_i\xi(\lambda x)$

$\mathbf{I}$ = unit diagonal matrix.

The diagonal elements g_{ii} and the off-diagonal elements g_{ij} of matrix $\mathbf{G}$ are

$$g_{ii} = \frac{1}{D_{(i)}}\left[\Sigma_{r(i)} - \frac{\chi_{(i)}}{k_{eff}}\nu_{(i)}\Sigma_{f(i)}\right]$$

and

$$g_{ij} = -\frac{1}{D_{(i)}}\left[\frac{\chi_{(i)}}{k_{eff}}\nu_{(j)}\Sigma_{f(j)} + \Sigma_{s(j\to i)}\right] \ . \tag{5}$$

A nontrivial solution exists only when the determinant of $(\mathbf{G} - \lambda^2\mathbf{I})$ is zero; hence, the λ^2 are calculated as the roots λ_j^2 of the characteristic polynomial of the matrix $\mathbf{G}$. For each λ_j^2 value, the α_i are the elements of the corresponding eigenvector. A general solution of the diffusion equation is

$$\boldsymbol{\Phi} = \mathbf{H}\cdot\mathbf{F}_a\cdot\boldsymbol{a} + \mathbf{H}\cdot\mathbf{F}_b\cdot\boldsymbol{b} \ , \tag{6}$$

where

$\mathbf{H}$ = matrix constructed from the eigenvectors of $\mathbf{G}$

$\mathbf{F}_a$ = diagonal matrix, the elements of which are the odd components $f_a(\lambda_i x)$ of the eigenfunctions.

Similarly, elements $f_b(\lambda_i x)$ of the diagonal matrix $\mathbf{F}_b$ are the even components of the eigenfunctions. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ with elements a_i and b_i are coefficients determined from the boundary conditions.

In slab geometry, the eigenfunctions are hyperbolic, trigonometric, or a combination of both, depending on whether the λ_i values are purely real, imaginary, or complex. In circular geometry, the eigenfunctions are the Bessel functions.

Define vector $\boldsymbol{q}$ with elements $\phi_{b(g)}$, which are the flux values on the boundary at ($x = \pm x_b$ for slabs and $x = x_b$ for circular regions) taken from the reference solution. In slab geometry, the same flux value is assumed on both boundaries. Since symmetric boundary conditions are imposed, the coefficients a_i of the odd components vanish. The coefficients b_i are easily calculated from

$$\mathbf{H}\cdot\mathbf{F}_b\cdot\boldsymbol{b} = \boldsymbol{q} \ . \tag{7}$$

For a reflector in slab geometry with a zero-flux external boundary condition, the procedure is very similar. The boundary condition on the inner boundary $\phi_{b(g)}$ at ($x = -x_b$) is applied, where x_b is the reflector half-thickness. Note that the equation for the coefficients b_i and the derived expression for the average flux remain the same because

$$a_i f_a(\lambda_i x_b) = -b_i f_b(\lambda_i x_b) \ , \tag{8}$$

which follows from the zero-flux boundary condition. More care is required in deriving the expression for the neutron current.

The solution procedure for circular regions (and slab geometry) is exact. In the case of square regions, the leakage is equal in each direction. By integration of the solution in the transverse direction, it can be shown that through the separability assumption, the transverse leakage component degenerates into an effective transverse buckling B_y^2. Using the relation

$$\tilde{D}_g B_y^2 \tilde{\phi}_g = \tfrac{1}{2}L_g \ , \tag{9}$$

where L_g is the total leakage rate from group g, the transverse buckling can easily be calculated, since L_g is known from the reference solution and must be reproduced exactly by the equivalent homogeneous diffusion solution. Rectangular and three-dimensional regions can be treated in a similar manner.

The implicit albedo boundary conditions for the equivalent homogeneous cell are effectively obtained from the average flux and current on the cell boundary, as calculated in the transport solution for the cell. The net current is calculated from the net leakage. For group g, the boundary condition in the diffusion approximation is

$$\frac{J_{b(g)}}{\Phi_{b(g)}} = \frac{1}{2}\left[\frac{1 - \alpha_{(g)}}{1 + \alpha_{(g)}}\right] \ , \tag{10}$$

where Φ_b and J_b are the average boundary flux and net current, respectively, and $\alpha_{(g)}$ is the group albedo.

For fuel assembly homogenization, the albedo boundary conditions are defined directly by the critical albedo search. At present, the albedo is assumed to be group independent.

III. APPLICATION TO FUEL ASSEMBLY HOMOGENIZATION

To test the proposed homogenization methods, the Koebke benchmark[4] was analyzed. It was used to compare the proposed method with the SET method and to compare the nodal transport and the nodal diffusion solutions in a fuel assembly.

TABLE I

Four-Group Cross Sections for the Koebke PWR Benchmark Problem*

Parameter	g	F1	F2	CW	CR	SS	RW
$\nu\Sigma_{f(g)}$ (cm^{-1})	1	0.00812	0.00812	---	---	---	---
	2	0.0006	0.000696	---	---	---	---
	3	0.00792	0.00912	---	---	---	---
	4	0.0888	0.1248	---	---	---	---
$\Sigma_{a(g)}$ (cm^{-1})	1	0.0039	0.0039	0.00024	0.0017	0.001	0.00035
	2	0.00225	0.00229	0.000016	0.0077	0.00077	0.000026
	3	0.0223	0.0228	0.0015	0.099	0.0082	0.0013
	4	0.07	0.086	0.027	0.54	0.11	0.028
$\Sigma_{(g\to g+1)}$ (cm^{-1})	1	0.061	0.061	0.078	0.072	0.0026	0.084
	2	0.061	0.061	0.1	0.055	0.0034	0.11
	3	0.062	0.061	0.099	0.049	0.0031	0.13
$D_{(g)}$ (cm)	1	2.6	2.6	3.3	2.4	1.8	2.6
	2	1.1	1.1	1.2	1.1	0.94	1.4
	3	0.84	0.84	0.71	0.63	0.38	0.84
	4	0.35	0.35	0.26	0.21	0.36	0.24
$\chi_{(g)}$	1	0.7517	0.7517	---	---	---	---
	2	0.2483	0.2483	---	---	---	---

*F1 and F2 are low- and high-enriched fuel, respectively; CW is the water-filled guide thimble; CR is the control rod; SS is the stainless steel core shroud; and RW is the water reflector. Fission neutrons appear in groups 1 and 2 only. The number of neutrons per fission ν is 2.8 in group 1 and 2.4 in other groups.

The Koebke PWR benchmark is a simplified PWR core with fuel assemblies of 12-cm side length and 8×8 channel array, four channels containing guide tubes. The guide tubes contain control rods or else they are filled with water. The core is surrounded by a 1.5-cm-thick stainless steel shroud. The total thickness of the reflector region is 12 cm. The geometry of the core and of the fuel assemblies is taken from Ref. 4 and is shown in Fig. 1. The four-group cross sections for different core constituents are given in Table I.

The four-group cross-section set was applied to generate a full-core heterogeneous solution in the diffusion approximation by GNOMER, which is a Green's function nodal diffusion code,[11] using solution methods described in Ref. 12. The solution over individual fuel assemblies was also obtained with GNOMER, and a critical albedo search was applied on the boundary conditions. Condensation to two groups and homogenization by the EDH method were performed to obtain the effective diffusion parameters for each type of assembly. Reflector constants were derived for a configuration similar to that used by Koebke[4]: The row of assemblies across the core diameter was considered, water channels and control rods were ignored, and the calculation was performed in one dimension (ignoring leakage in the y direction). The assembly homogenized cross sections are given in Table II.

The results of the core power distribution calculations are presented in Fig. 2. Coarse-mesh results were calculated with the GNOMER code, and they correspond to one mesh per assembly and third-order flux expansion for integration purposes. Convergence criteria were selected so that the iteration error is negligible. Only the results for the rodded core are given as these were more restrictive in terms of accuracy than those for the unrodded core.

When the heterogeneous diffusion solution was compared with the reference, which was calculated with a nodal transport method, the agreement was very

TABLE II

Assembly Cross Sections for the Koebke Benchmark Problem Averaged by the EDH Method

	g	$D_{(g)}$	$\Sigma_{a(g)}$	$\nu\Sigma_{f(g)}$	$\Sigma_{(g\to g+1)}$	$\mu_{(g)}$
FL	1	1.50400	0.0080867	0.0047576	0.01873042	0.9983
	2	0.35629	0.0694918	0.0857355	---	1.0361
FH	1	1.52340	0.0081108	0.0051284	0.01799933	1.0006
	2	0.36045	0.0858192	0.1216174	---	1.0490
FC	1	1.43368	0.0100475	0.0046210	0.01805838	0.9868
	2	0.30619	0.0789324	0.0757924	---	0.8890
R[a]	1	1.02020	0.0014103	---	0.03087685	1.0827
	2	1.21120	0.1004083	---	---	2.7003

[a]The flux- and volume-weighted two-group reflector diffusion constants are 1.42065 and 0.25344, respectively.

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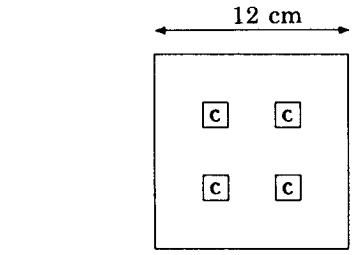
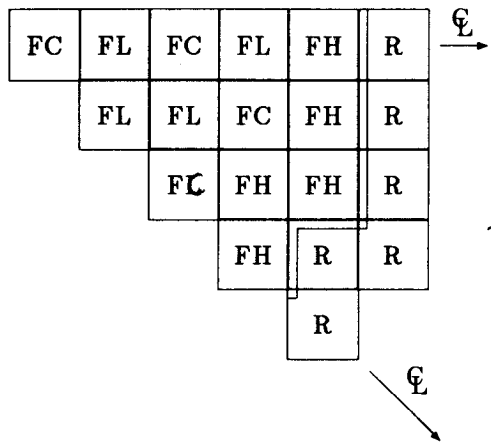


(a) Fuel assembly geometry



(b) PWR core octant geometry

Fig. 1. Koebke PWR benchmark geometry: (a) structure of the 8 × 8 fuel assembly with four channels that contain either control rods or water and (b) the core octant, where FL and FC are unrodded and rodded low-enriched fuel assemblies, FH is highly enriched unrodded assemblies, and R is the reflector, including the stainless steel core shroud.

Average power distribution
(reference and % differences)

1.356 -0.3 -0.6 -0.2	1.601 -0.1 -0.7 -0.9	1.103 -0.3 -0.6 -0.1	1.030 -0.1 -0.4 -0.3	0.830 +0.2 -0.4 -0.1
	1.575 +0.0 -0.5 -0.9	1.299 -0.1 -0.5 -0.6	0.811 -0.1 +0.4 +0.9	0.769 +0.2 -0.2 +0.2
		0.921 -0.1 +0.4 +0.6	1.059 +0.0 +0.9 +0.4	0.618 +0.3 +0.6 +0.6

k_{eff}

a	0.90384
b	0.90273
c	0.90614
d	0.90269

0.742
+0.2
+1.5
+0.8

Fig. 2. Koebke PWR benchmark results for the rodded core. The entries are as follows: (a) reference transport solution by Koebke, (b) heterogeneous fine-mesh diffusion solution by the GNOMER code, (c) coarse-mesh diffusion solution using SET homogenized cross sections by Koebke, and (d) coarse-mesh diffusion solution using EDH homogenization and the GNOMER code.

good. This is rather surprising as the same cross-section set was used for both the diffusion and the transport calculations. In a realistic application, the cell homogenized transport equivalent cross sections should be obtained by a procedure such as Hébert's SPH method[6] so that a transport calculation with homogenized cells would be equivalent to an explicit heterogeneous transport calculation over a fuel assembly. In principle, equivalent cell cross sections for a diffusion calculation over an assembly are not necessarily the same, but the results show that the differences are small (equivalent diffusion cell cross section is discussed in Sec. IV). The results indicate that the use of the diffusion theory for typical PWR fuel assembly calculations does not introduce a significant error into the global results, provided that a fully converged diffusion solution in multigroup form is used. From experience, a two-group treatment in a fuel assembly is grossly inadequate.

The coarse-mesh power distribution was not reported by Koebke,[4] so it was generated from the published SET cross sections with the GNOMER code. The k_{eff} and the maximum relative error in the average assembly power differ very slightly from the published results, but the differences are negligible.

The two-group EDH cross sections for different fuel assemblies were prepared as described earlier. Global calculation was performed by using GNOMER. In spite of the additional simplifications in cross-section homogenization, the results of the EDH method are comparable with those using SET cross sections, and they agree well with the reference results.

IV. APPLICATION TO CELL HOMOGENIZATION

The EDH method was also applied to unit cell homogenization problems. A typical PWR fuel assembly was considered.

To test cell homogenization methods, it would be ideal to perform a detailed multigroup transport calculation of a fuel assembly in full heterogeneous geometry. Unfortunately, the WIMS/D-4 lattice code,[13] which we use for lattice calculations, cannot handle problems of such complexity because of numerical and computational limitations. Published results, which could be used as reference, are usually restricted to two groups, which we consider inadequate. Therefore, a geometrically simpler test case was considered.

A patch of unit cells was assumed that contained a 3 × 3 array of cells with white boundary conditions, representing approximately a cutout of a fuel assembly. The geometry is shown schematically in Fig. 3. The

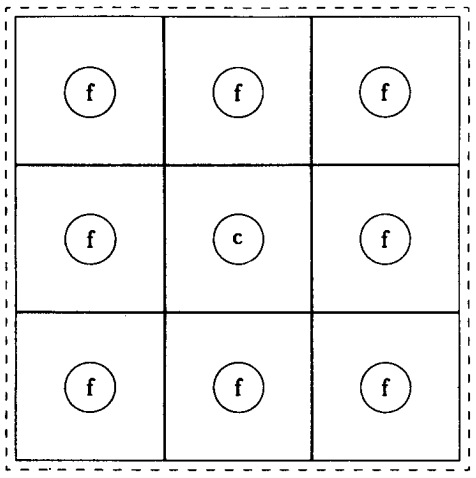


Fig. 3. Lattice cell array configuration. The peripheral cells all contain fuel (f), while the central cell (c) may contain a fuel rod, water, a BPR, or a control rod. When a fuel rod is contained in the central cell, an extra water region is added on the outside (dashed line).

outer cells always contain fuel rods, while the central cell may contain a fuel rod, water, a burnable poison rod (BPR), or a control rod. To preserve the average fuel-to-moderator ratio of a real assembly (due to water channels), an extra layer of water was added in some cases. The cell patches were as follows:

1. fuel cell array with an extra water region on the outside
2. array with a water-filled channel in the center
3. array with a BPR (boron glass type) in the center
4. array with a control rod (Ag-In-Cd type) in the center.

The dimensions and material composition of a real PWR with 2.6 wt% enriched fuel were taken. Since this test case is not intended to be used as a benchmark, the detailed geometry (i.e., gap and cladding thickness, spacer grids, guide tubes, etc.) and material composition are not specified.

Reference calculations for the patches of cells containing different types of cells in the center were performed with the explicit two-dimensional collision probability method of WIMS (the so-called PIJ transport option) in a 32-group approximation. For the central cell, the volume average group flux (i.e., the cell spectrum) and average cross sections according to the FVH method for each type of cell were read from the cell edit of the WIMS output. The diffusion constant was defined as $1/3\Sigma_{tr}$, where Σ_{tr} is the cell average transport cross section. Condensation to six groups with the cell spectrum was performed. The central cell boundary flux was approximated by the average flux of the outermost region of the cell. The multiplication factor was read from the leakage edit of WIMS. This information was sufficient to perform EDH correction of the six-group cross-section set. The method could be applied to the full 32-group cross-section set, but a parametric study on a real PWR-type assembly showed that a six-group data set introduces a relatively small error, with the benefit of a significant saving in the processing time.

To test the EDH method for cell homogenization, equivalent patches of homogenized cells were set up. Homogenized cell cross sections prepared by the FVH and EDH methods were used. Calculations in the six-group diffusion approximation and with reflective boundary conditions on the outer boundary of the patch were made with the GNOMER code. The values of the multiplication factors were compared with the reference 32-group collision probability (PIJ) calculations. The results are presented in Table III. The FVH and EDH methods are considered. The effect of simplified geometry transport solution (the so-called DSN transport option in WIMS) to obtain homogenized cross sections for the central cell is also considered (labeled EDH/DSN).

The homogenized fuel cell cross sections were always taken from the central cell transport solution of patch 1.

The diffusion solution over patch 1 is almost an infinite-medium problem because the heterogeneity due to a thin water layer on the outside is negligible and the cells are homogeneous by definition. Good agreement with the reference k_{eff} value shows that condensed and homogenized diffusion parameters of the unit cells were derived consistently.

Patch 2 represents a water gap at the center of a fuel cell array. From a global point of view, this heterogeneity is not very strong since the change in k_{eff} compared with a complete array of fuel cells remains below 0.7% $\Delta k/k$. The small differences in k_{eff} in the homogenized diffusion solution can be attributed to inappropriate fuel cell cross sections, which correspond

TABLE III

Errors in Multiplication Factor for Patches of Cells from Equivalent Diffusion Calculations and Different Homogenization Options Compared with Explicit Collision Probability Calculations

Patch	Transport k_{eff}	Diffusion ($\Delta k/k \times 10^5$)		
		FVH	EDH	EDH/DSN
1	1.11928	+18	+6	−28
2	1.11176	−71	−163	−195
3	0.84957	−1964	−23	−175
4	0.59723	−9521	−810	−771

to patch 1 with a much larger overall fuel-to-moderator ratio. The differences are not physically significant.

Patch 3 and particularly patch 4 contain strong absorbers at the center. The performance of the EDH method is quite remarkable, considering the simplicity of the six-group diffusion calculation compared with the reference 32-group transport solution. The homogenization error is reduced by an order of magnitude if EDH is applied instead of the simple FVH method.

In practice, an explicit two-dimensional collision probability calculation is excessively time-consuming. To reduce computation costs, the central cell is modeled explicitly, but the surrounding fuel cells are treated in the so-called supercell approximation by smearing them into equivalent concentric rings (this is done automatically by WIMS). The resulting configuration in one-dimensional radial geometry can be solved effectively by the S_n method, which is also available in WIMS (i.e., the DSN transport option). The results in Table III (labeled EDH/DSN) indicate that the additional error introduced by the simplified transport treatment in deriving the equivalent homogenized cross sections is minimal, while the computational time is reduced by an order of magnitude.

For PWR core calculations, it is proposed that a simplified transport (DSN) calculation and the EDH method be used to obtain few-group cell cross sections. A multigroup diffusion theory calculation over a fuel assembly with a critical albedo search and the EDH method can then be applied to generate assembly-averaged two-group cross sections to be used for global reactor analysis.

V. COMBINED EFFECTS ON GLOBAL REACTOR ANALYSIS

To analyze the effects on real PWR cores, some test calculations were done for the Krško nuclear power plant cycle-1 core [a Westinghouse 600-MW(electric) reactor]. The core was loaded with three types of fuel: 2.1, 2.6, and 3.1 wt% enrichment, respectively. Some assemblies were loaded with BPRs in clusters of 8 or 12. The core loading is shown in Fig. 4, which represents a core octant.

The simplified 32-group transport calculation (DSN) was used for unit-cell calculations. Various homogenization options were tested for preparing six-group unit cell cross sections (for fuel rods, water gaps, BPRs, and

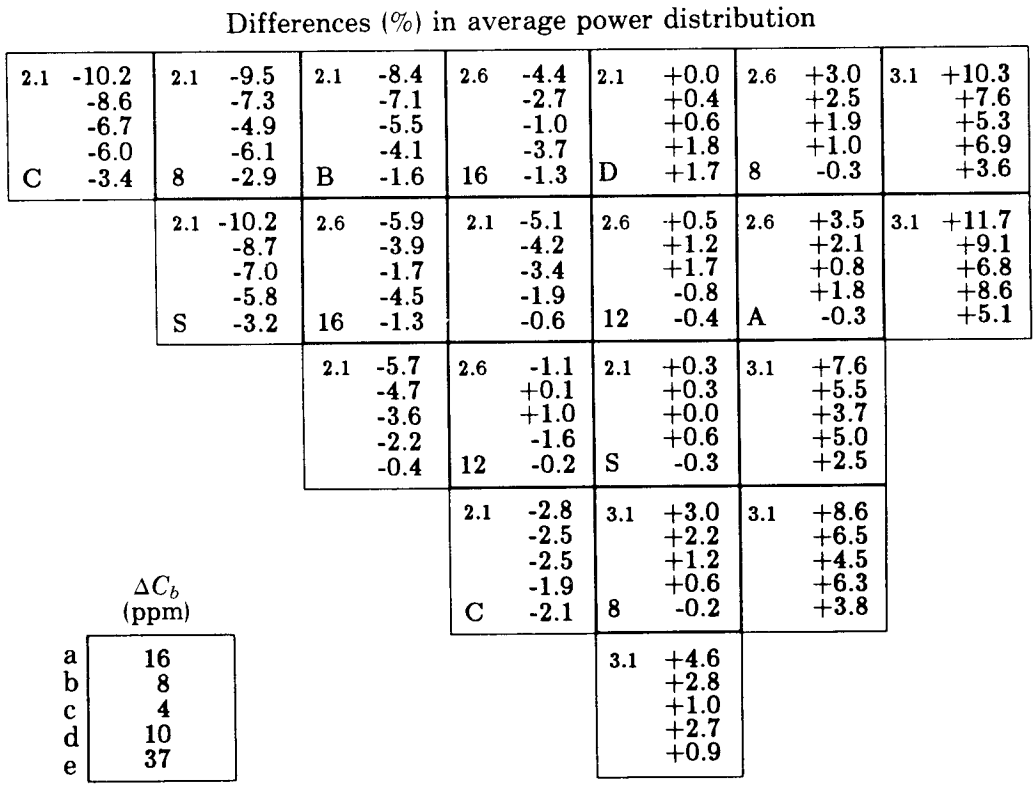


Fig. 4. Influence of different homogenization options on PWR core critical boron concentration and power distribution. On the left edge, the enrichment (weight percent), the control rod locations (A, B, C, D, or S), and the number of BPRs (if any) are given. The reference power distribution results are flux-map measurements at hot-zero-power conditions. The entries are as follows: (a) no criticality search, (b) critical buckling search, (c) critical albedo search, (d) EDH on assembly cross sections and critical albedo search, and (e) EDH on cell and assembly cross section and critical albedo search.

control rods), and two-group assembly-averaged cross sections for global reactor analysis. In assembly calculations, one node per cell was used in GNOMER, and second-order polynomial flux expansion for integration was found to be adequate. In global reactor analysis with GNOMER, one node per assembly was used, with ten discrete regions in the axial direction and third-order polynomial flux expansion for integration.

The calculated results were compared with the measured data and are presented in Fig. 4. The calculated results are not best-estimate design values because for better comparison, the same reflector constants were used in all cases. They were obtained from some older analyses and were not position dependent. The reflector constants and the WIMS multigroup library are considered to be the major sources of systematic errors in the current calculations.

Measurements were performed at 4% power with the D-bank of control rods 17% inserted into the core. These effects were not accounted for in the calculations. Also, at symmetric locations in the core quadrants, differences of up to 10% in the measured power were observed, particularly on the core periphery, where the power level is relatively low. It is estimated that errors in the measurements amount to ~5%.

Different homogenization options were considered. In row 1 in Fig. 4, the differences in predicted critical boron concentration and power distribution are displayed when no criticality search is performed and FVH cross sections are used.

The differences in row 2 were obtained with a critical buckling search. The results show that the overall leakage from the core increases, thus lowering the predicted critical boron concentration and causing the power distribution to be more peaked at the center. The change in average power is larger in the assemblies containing strong absorbers. The critical buckling search mainly affects the results through spectral changes since the flux distributions in assembly calculations are hardly affected. Although the criticality search effect was negligible in the Koebke PWR benchmark, its effect in a real application is much more pronounced.

For the differences in row 3, the critical albedo search was performed, so that radial leakage effects were treated explicitly. Comparison with the previous test in row 2 shows that flux redistribution effects due to radial leakage are at least as important as the spectral effects alone. It can be concluded that an effective buckling may be adequate for representing the axial leakage, but it is physically unrealistic for modeling the leakage in the radial direction.

The effects of generating assembly-averaged cross sections with the EDH method are shown in row 4. The assembly average power decreases in assemblies containing strong absorbers (i.e., BPRs). This decreases the importance of strongly absorbing regions, so the overall core reactivity increases and so does the predicted critical boron concentration.

The effects of EDH on cell cross sections is much stronger, as seen from the results in row 5. It causes a significant increase in reactivity of assemblies containing strong absorbers (as seen from the previous results) and hence the overall reactivity of the core, but the average assembly power, compared with the nearest neighbors, is affected to a smaller extent. A considerable increase in the critical boron concentration is observed, and the power distribution becomes more peaked at the center.

The EDH method was also applied in the calculation of a rodded core to calculate the change in reactivity due to the insertion of control rods. The results are presented in Table IV. The difference between the FVH and EDH methods in generating the cross sections is 16% in control rod worth. Although the calculated results are not best estimate, the results using the EDH method agree well with the measurements. The accuracy of the control rod worth measurements is estimated to be ±5%.

VI. SUMMARY OF RESULTS

VI.A. Fuel Assembly Cross-Section Homogenization

The EDH method is a special case of the SET method where symmetric boundary conditions are assumed. Since we are usually interested in core configurations that are approximately critical, $k_{eff} = 1$ is implied, and the albedo is adjusted until the criticality condition is satisfied. The method could be refined by defining criteria to obtain group-dependent albedoes that would satisfy the criticality condition, but this possibility has not been investigated further.

The EDH method for fuel assembly cross-section homogenization produces favorable results, especially if we consider the following:

1. No new parameters are introduced.

2. Standard diffusion codes without modifications are applicable.

3. The cross sections are environment independent.

4. The additional computational cost of EDH correction to the cross sections is practically negligible.

VI.B. Cell Cross-Section Homogenization

It has been demonstrated that the EDH method for generating homogenized cross sections works well for fuel assemblies as well as for individual cells within an assembly. In comparison with the reference two-dimensional transport solution, the equivalent diffusion solution was capable of predicting the change in reactivity to within 0.01 $\Delta k/k$ even in the extreme case of a control rod where the overall change in reactivity was $-0.5\ \Delta k/k$.

TABLE IV

The Effect of Different Homogenization Options on Control Rod Worth*

Method	A + B + C + D ($\Delta k/k \times 10^5$)	A + B + C + D + S ($\Delta k/k \times 10^5$)
FVH	---	11 062
EDH	5495	9 533
Measured	5234	---

*Control rod banks A, B, C, D, and S were considered.

VI.C. Combined Effects on Global Reactor Analysis

The results show that leakage in the radial direction must be accounted for explicitly. The derivation of equivalent homogenized diffusion cross sections for fuel assemblies as well as for the cells within an assembly is crucial for accurate predictions of power distributions and reactivity changes in PWR cores. In fact, homogenization procedures seem to be more important than transport effects, provided that at least six to ten groups are used in assembly calculations. Under such circumstances, standard diffusion codes can be applied for assembly calculations as well as for global reactor analysis without significant loss of accuracy, and transport theory methods are restricted to lattice cell calculations.

The proposed EDH method provides a simple tool for deriving few-group homogenized constants for standard diffusion codes with a negligible additional computational effort.

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