SKLAPLJANJE VRTILNIH KOLIČIN

(GRADNJA LASTNIH STANI IN SKLOPLJENIH OPERATORJEV)

Pri večini realnih sistemov je treba upoštevati več vrtilnih količin: Vsak e ima tirno in spin sko vrtilno količino, ki sta sklopljena preko interakcije spin-tir in v več elektronskih sistemih se vrtilne količine sklopljajo na različne načine, odvisno od situacije, ki jo opisujemo.

- SKLOPITEV DVEH VRTILNIH KOLIČIN

\[
\begin{align*}
\hat{J}^2 (2) | j_2 m_2 \rangle &= j_2 (j_2 + 1) | j_2 m_2 \rangle \\
\hat{J}^z (2) | j_2 m_2 \rangle &= m_2 | j_2 m_2 \rangle \\
\hat{J}^2 &= \hat{J}^2 (1) + \hat{J}^2 (2) \\
\hat{J} &= \sqrt{\hat{J}^2 (1) + \hat{J}^2 (2)}
\end{align*}
\]

Komponente vrtilnih količin neodvisnih podsystemov med seboj komutirajo:

\[
[\hat{J}_x (1), \hat{J}_x (2)] = 0 \rightarrow [\hat{J}_x, \hat{J}_y] = i\hat{J}_z
\]

\[
\hat{J} \text{ ima enake lastnosti kot } \hat{J} (1) \text{ in } \hat{J} (2); \text{ je vrtilna količina}
\]

\[
\begin{align*}
\hat{J}^2 | (j_1 j_2) JM \rangle &= J (J + 1) | (j_1 j_2) JM \rangle \\
\hat{J}_z | (j_1 j_2) JM \rangle &= M | (j_1 j_2) JM \rangle
\end{align*}
\]

Lastna funkcija \( J^2 \) in \( J_z \)

Produkt enodelčnih valovnih funkcij \( | j_1 m_1 j_2 m_2 \rangle = | j_1 m_1 \rangle | j_2 m_2 \rangle \)

je lastna funkcija \( J_z \); \( J_z | j_1 m_1 j_2 m_2 \rangle = (\hat{J}_z (1) + \hat{J}_z (2)) | j_1 m_1 j_2 m_2 \rangle \)

ne pa \( J_x \).

\[
\hat{J}_z = (m_1 + m_2) | j_1 m_1 j_2 m_2 \rangle
\]

To konstruiramo z linearo kombinacijo produktnih funkcij, tako da je \( M = m_1 + m_2 \), z vsak čas.
\[ |(j_1 j_2)JM\rangle = \sum_{m_1,m_2} |j_1 m_1, j_2 m_2\rangle \langle j_1 m_1, j_2 m_2 |JM\rangle \quad \text{v teži} \]

\[ \langle j_1 m_1, j_2 m_2 |JM\rangle = \langle j_1 m_1, j_2 m_2 |(j_1 j_2)JM\rangle \quad \text{Clebsch–Gordanov} \]

ker tvorijo produktne funkcije \( |j_1 m_1\rangle |j_2 m_2\rangle \) ortonormalni sest.

Absolutna faza koeeficientov CG je dobrodoljena z dogovorom, pri danih \( j_1 \), \( j_2 \) in \( J \) ter maksimalni vrednosti \( m_1 \) in \( M \) (se pravi \( m_1 = j_1, M = J \)) je koeeficient realen in nenegativen.

\[ \Rightarrow \text{Vsi CG so REALNI.} \]

\[ \text{- INVERZNA TRANSPORMACIJA:} \]

\[ |j_1 m_1, j_2 m_2\rangle = \sum_{JM} |(j_1 j_2)JM\rangle \langle JM |j_1 m_1, j_2 m_2\rangle \]

\[ \langle JM |j_1 m_1, j_2 m_2\rangle = \langle (j_1 j_2)JM |j_1 m_1, j_2 m_2\rangle = \langle j_1 m_1, j_2 m_2 |JM\rangle * \]

\[ \Rightarrow \langle j_1 m_1, j_2 m_2 |JM\rangle = \langle JM |j_1 m_1, j_2 m_2\rangle \]

\[ \text{PRIMER: KAKŠNA SO LASTNA STANJA SKUPNE VRHTLINE KOLIČINE \( J \) \& 2 ELEKTRONI S TIRNO VRHTLINNO KOLIČINO \( \ell \)?} \]

\[ \ell_1 = 1, \ell_2 = 1 \quad |1m_1, 1m_2\rangle \quad m_1, m_2 = -1, 0, 1 \]
$|1_{11},1_{11}\rangle \rightarrow M_L = m_1 + m_2 = 2$ edino stanje $J = 2$

$|1_{11},2_{2}\rangle = |1_{11},1_{11}\rangle$ ostala stanja z $J = 2$ dobimo z operatorjem $\hat{L}_- = \hat{L}_{(1)} + \hat{L}_{(2)}$

$|1_{11},2_1\rangle = \frac{4}{12}(1_{11},1_{10}\rangle + 1_{10},1_{11}\rangle)$

$|1_{11},2_0\rangle = \frac{4}{16}(1_{11},1_{-1}\rangle + 21_{10},1_{10}\rangle + 1_{1-1},1_{11}\rangle)$

$|1_{11},2_{-1}\rangle = \frac{4}{12}(1_{10},1_{-1}\rangle + 1_{1-1},1_{10}\rangle)$

$|1_{11},2_{-2}\rangle = |1_{-1},1_{-1}\rangle$

Obstaja še ena linearna kombinacija $a|1_{11},10\rangle + b|1_{10},11\rangle$, ki imajo $M_L = 1$, ta ustreza $J = 1$. Ortogonalnost $|1_{11}2_{1}\rangle$ in $|1_{11}1_{1}\rangle$ prideve do $a + b = 0$.

Normalizirano stanje $|1_{11}1_{1}\rangle = \frac{1}{2}(1_{11},1_{0}\rangle - 1_{10},1_{1}\rangle)$. Ostala $J = 1$ stanja z aplikacijo $\hat{L}_-$. Podobno dobimo še stanje $J = 0$: $|1_{11}0_{0}\rangle$, za katero zahtevamo, da je ortogonalno na $|1_{11}2_{0}\rangle$ in $|1_{11}1_{0}\rangle$.

Očito lahko konstrukcijski pristop posplošimo.

Wigner 1931: $\hat{J}_+ (j_1, j_2) NN = \hat{J}_+ \sum_{m_1, m_2} j_1 m_1 j_2 m_2 \langle j_1 m_1 | j_2 m_2 \rangle NN = 0$

prideve do rekurzijskih rešitev med CG in izraza za CG koeficient.
\[ \langle j_1 m_1, j_2 m_2 | J M \rangle = S(m_1 + m_2 | M) \frac{(2J+1)(j_1 + j_2 - J)!(j_1 - m_1)!(j_2 - m_2)!(J+M)!(J-M)!}{(j_1 + j_2 + J + 1)!(J + j_1 - j_2)!(J + j_2 - j_1)!(j_1 + m_1)!(j_2 + m_2)!} \]

\[ \times \sum_r (-1)^{j_1 - m_1 + r} \frac{(j_1 + m_1 + r)! (j_2 + J - m_2 - r)!}{r! (J - M - r)! (j_1 - m_1 - r)! (j_2 + J + m_2 + r)!} \]

- ŠE EN PRIMER: VRTILNA KOLIČINA ELEKTRONA, SESTAVLJENA IZ TIRNE IN SPINŠKE.

\[ \mathbf{j} = \mathbf{l} + \mathbf{s} \]

\[ \lambda = \frac{1}{2} \]

<table>
<thead>
<tr>
<th>J</th>
<th>m_2</th>
<th>\frac{1}{2}</th>
<th>-\frac{1}{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>l + \frac{1}{2}</td>
<td>\sqrt{\frac{l + m_j + \frac{1}{2}}{2l+1}}</td>
<td>\sqrt{\frac{l - m_j + \frac{1}{2}}{2l+1}}</td>
<td></td>
</tr>
<tr>
<td>l - \frac{1}{2}</td>
<td>\sqrt{\frac{l - m_j + \frac{1}{2}}{2l+1}}</td>
<td>-\sqrt{\frac{l + m_j + \frac{1}{2}}{2l+1}}</td>
<td></td>
</tr>
</tbody>
</table>

\[ |(l + \frac{1}{2}, j_1, m_j)\rangle = \sqrt{\frac{l + m_j + \frac{1}{2}}{2l+1}} |l m_j - \frac{1}{2}, \frac{1}{2}\rangle + \sqrt{\frac{l - m_j + \frac{1}{2}}{2l+1}} |l m_j + \frac{1}{2}, -\frac{1}{2}\rangle \]

\[ |(l - \frac{1}{2}, j_1, m_j)\rangle = \sqrt{\frac{l - m_j + \frac{1}{2}}{2l+1}} |l m_j - \frac{1}{2}, \frac{1}{2}\rangle - \sqrt{\frac{l + m_j + \frac{1}{2}}{2l+1}} |l m_j + \frac{1}{2}, -\frac{1}{2}\rangle \]
ŠE NEKAJ LASTNOSTI CG:
\[
\begin{align*}
\langle j_2 m_2 j_1 m_1 \mid J M \rangle &= (-1)^{j_1 - j_2 - J} \langle j_2 m_2 j_1 m_1 \mid J M \rangle \\
\langle J - M j_2 m_2 \mid j_1 m_1 \rangle &= (-1)^{j_2 + m_2} \frac{2j_1 + 1}{2J + 1} \langle j_1 m_1 j_2 m_2 \mid J M \rangle \\
\langle j_1 - m_1 j_2 - m_2 \mid J M \rangle &= (-1)^{j_1 + j_2 - J} \langle j_1 m_1 j_2 m_2 \mid J M \rangle
\end{align*}
\]

NI SIMEJKE MED CG ZA MENJAVO VRSTNEGA REDA TREH VRTILNIH KOLICI.
ZATO OBičAJNO RAJE UPORABLJAMO WIENERJEV 3-G SIMBOL:
\[
\begin{align*}
(j_1, j_2, j_3) &= (-1)^{j_1 - j_2 - m_3} \frac{\langle j_1 m_1 j_2 m_2 \mid j_3 - m_3 \rangle}{\sqrt{2j_3 + 1}} \\
\langle j_1 m_1 j_2 m_2 \mid j_3 - m_3 \rangle &\neq 0 \quad m_1 + m_2 + m_3 = 0, \quad |j_1 - j_2| \leq j_3 \leq j_1 + j_2
\end{align*}
\]

LAJTNOSTI 3-G SIMBOLOV:
\[
\begin{align*}
(j_1, j_2, j_3) &= (j_2, j_3, j_1) = (j_3, j_1, j_2) \\
(j_1, j_2, j_3) &= (-1)^{j_1 + j_2 + j_3} (j_1, j_2, j_3) \\
(j_1, j_2, j_3) &= (-1)^{j_1 + j_2 + j_3} (j_1, j_2, j_3)
\end{align*}
\]

SOPA PERMUTACIJA
NE SPREMENI VREDNOSTI,
LIHA PA UVEDE FAZNI
FAKTOR
KER LASTNA STANJA TVORIJO ORTONORMALNI SET:

\[ \langle j_1 m_1, j_2 m_2 | j_3 m_3 \rangle \langle j_3 m_3 | j_1 m_1, j_2 m_2 \rangle = \delta (m_1, m_1) \delta (m_2, m_2) \]

\[ \langle j_1 j_2 | j_3 m_3 \rangle \langle j_1 j_2 | j_3 m_3 \rangle = \delta (j_1, j_1) \delta (j_2, j_2) \delta (m_3, m_3) \]

VELJajo še izreke MED EZITOMA 3-j SIMBOLI:

1. \[ \sum_{j_3 m_3} \langle j_1 m_1, j_2 m_2 | j_3 m_3 \rangle \langle j_3 m_3 | j_1 m_1, j_2 m_2 \rangle = \delta (m_1, m_1) \delta (m_2, m_2) \]

2. \[ \sum_{j_3 m_3} \langle j_1 m_1, j_3 m_3 | j_2 m_2 \rangle \langle j_2 m_2 | j_1 m_1, j_3 m_3 \rangle = \delta (j_1, j_2) \delta (m_3, m_3) \]

\[ \delta (j_1, j_2) \delta (m_3, m_3) = \frac{1}{\sqrt{2j_3 + 1}} \]

SKLOPITEV SFERNIH TENZORSKIH OPERATORJEV (STO); TENZORSKI PRODUKT STO \( X^K \) S KOMPONENTAMI \( X_Q^K \)

\[ X_Q^K = \{ f^Q_{2^k_1} (1) \} f^Q_{2^k_2} (2) \}

\[ \sum_{k_1, k_2} f^Q_{2^k_1} (1) \mu^Q_{k_2} (2) \langle k_1 | q_1, q_2, q_2 | k_2 \rangle \]

\( \mu_{2^k_1, 2^k_2} \) deluje na različne prostornine stopnje sistema, recimo (1) na spinški del, (2) na tirični del elektrona, ali v dvoelektronskem sistemu \( f^Q_{2^k_1} (1) \) deluje na koordinato prvega, \( \mu^Q_{2^k_2} (2) \) pa drugega elektrona.
Operator $\hat{X}_a^k$ se pri rotacijah transformira kot stanje vrtilne količine $|2,1_{k_2}; 1_k\rangle$ in je torej STO reda $K$. $\hat{X}_a^k$ je tenzorski produkt operatorjev $\hat{X}_a$ in $\hat{M}_z$, ki sta tenzorska operatorja reda $2$, in $\hat{X}_z$ in se transformirata kot stanje vrtilne količine $|2,1_{k_2}; 1_k\rangle$ in $|2,1_{k_2}; 1_k\rangle$.

Dva tenzorska operatorja istega reda lahko sestavimo v tenzorski produkt reda 0 (skalarni operator).

$$\hat{X}_0 = \{\hat{t}_0^2, \hat{t}_0^2\} = \sum_{q} \frac{t_0^2(1) \hat{M}_z(2)}{\sqrt{2q+1}} |2,2-2,2; 100\rangle$$

$$= \sum_{q} \frac{(-1)^q}{\sqrt{2q+1}} \sum_{k} (-1)^{2q} t_0^2(1) \hat{M}_z(2)$$

$$= \frac{(-1)^{2q}}{\sqrt{2q+1}} \hat{\chi}_z \hat{M}_z$$

**Primer!!**

**Coulombska interakcija med 2 elektronoma:**

$$\frac{1}{r_{12}} = \sum_{q} \frac{v_{12}^2}{k} P_k(\cos \theta)$$

$$r_{12} = r_1 \leq r_2$$

$$r_{12} = \{r_1, r_2, \text{ sicer}\}$$
Adicijski izrek za sferne harmonike:

\[ P_2(\omega, \theta) = \frac{4\pi}{2^2+1} \sum_{\ell} Y_2^\ell(\theta, \phi) Y_2^{\ell*}(\theta, \phi) \]

Definiramo C-tenzor \( \hat{C}_1^\ell \) s komponentami \( \hat{C}_1^\ell = \sqrt{\frac{4\pi}{2^2+1}} Y_2^\ell(\theta, \phi) \):

\[ P_2(\omega, \theta) = \hat{C}_1^\ell \cdot \hat{C}_2^\ell \rightarrow \frac{1}{V_{12}} = \sum_{\ell \neq \ell'} \hat{C}_1^\ell \cdot \hat{C}_2^{\ell'} \]

Coulombska interakcija je invariantna na urtenje obeh \( \varepsilon \) hkrat. Zato mora biti \( \hat{C}_1^\ell(1) \) in \( \hat{C}_2^\ell(2) \) sklopljena tako, da formirata tensorjski produkt reda 0 \( \rightarrow \) skalarni operator

Wigner - Eckartov teorem

Obravnavali smo, če transformacijske lastnosti (sklopljenih) lastnih stanj vrtline količine in sto: obe skupini se transformirata enak način, pač tako kot velevajo ireducibilne reprezentacije rotacijskih grup.

Zadaj nas zanima učinek simultanega urtenja tensorjskih operatorjev in lastnih stanj v matičnih elementih. Izkazuje se, da je mogoče matične elemente sto faktorizirati na produkt dveh delov, od katereh je samo en del (trivialno) odvisen od magnetnih kvantnih števil.
\[ P(\omega) \hat{t}_q^z |\gamma j m\rangle = P(\omega) \hat{t}_q^z P^\dagger(\omega) |\gamma j m\rangle \]
\[ = \sum_{2', m'} \hat{t}_2^z \hat{D}_{2' 2}(\omega) |\gamma j m\rangle \hat{D}_{m' m}(\omega) \]

Transformacija kot \( \hat{D}_{2' 2} \otimes \hat{D}_{m' m} \)

direktni produkt

Funkeija \( \hat{t}_2^z |\gamma j m\rangle \) se torej transformira kot produktne funkcije \( |\gamma k q\rangle |\gamma 2 j m\rangle \).

\[ \phi(\beta JM) = \sum_{2'' m''} \hat{t}_2^z |\gamma j m\rangle \langle 2'' q j m |JM\rangle = \sum_{\gamma'' JM} \langle \gamma'' J |\gamma '' JM\rangle \]

Inverzna transformacija

\[ \hat{t}_2^z |\gamma j m\rangle = \sum_{JM} \phi(\beta JM) |JM| \hat{t}_2^z |q j m\rangle = \sum_{\gamma'' JM} \langle \gamma'' J |\gamma '' JM\rangle |JM| |\gamma'' q j m\rangle \]

\[ \langle \gamma'' J |\gamma'' q j m \rangle = c(\gamma'' J) |\gamma'' q j m \rangle = c(\gamma'' J) \sqrt{2j'' + 1} \]

\[ \langle \gamma j m |\hat{t}_2^z |\gamma' q j' m' \rangle = (-1)^{j'' - j} (-)^m q m \sqrt{2j'' + 1} c(\gamma'' J) \]

Reducirani matični element
PRIMER I ZEEMANOV EFERUT, INTERAKCIJA MAGNETNEGA MOMENTA ATOMA
S ŠIBKIM ZunanJIM MAGNETNJIM POSEM. Z-OSI VEDOLJE SMERI $\mathbf{B}$.

$$\hat{H}_m = -\frac{\alpha}{\hbar} \mathbf{B} = \mu_B B (\hat{L}_z + g_S \hat{S}_z),$$

MAGNETNI MOMENT JE POSLEDICA VRTIJNE KOLICINE ELEKTRONA

MAGNETNI MOMENT JE OBITRALNI ORBITALNI GIPOMAGNETNI DEJSTV

SPLINSKI DEL GIROMAGNETNO RAZMERJE

$$\Delta E_{JM} = \langle (LS)JM | \hat{H}_m | (LS)JM \rangle$$

$$\hat{H}_m = \mu_B B g_S \hat{S}_z$$

Landé-JeV G FAKTOR

ZA POLJUBNO KOMPONENTO $q$

$$\frac{\langle (LS)JM | \hat{J}_q | (LS)JM' \rangle}{\langle (LS)JM | \hat{J}_q | (LS)JM' \rangle} = \frac{\langle (LS)JM \hat{J}_q (LS)JM' \rangle}{\langle (LS)JM \hat{J}_q (LS)JM' \rangle}$$

PODLOBNO ZA SPIN

$$\langle (LS)JM \hat{S}_2 (LS)JM' \rangle = \langle (LS)JM \hat{S}_2 (LS)JM' \rangle$$

$$g_S = \alpha_L + g_S \alpha_S$$ (neodvisen od $M$), $\hat{J}_2 = \hat{S}_2 + \hat{L}_2 \rightarrow \alpha_S + \alpha_L = 1$
Magnetna interakcija razepliva SLJ na podnivoje, ki so enakomerno razporejeni. Njihov razmak narašča sorazmerno z velikostjo \( B \) in je odvisen od SLJ preko Landé-Jevega \( g \) faktorja.

Izračunamo \( g_j \):

\[
\hat{L}^2 = (\hat{J} - \hat{S})^2 = \hat{J}^2 + \hat{S}^2 - 2 \hat{J} \cdot \hat{S} = \hat{J}^2 + \hat{S}^2 - 2 \hat{d}_S \hat{J}^2
\]

\[
\langle (LS)JM | \hat{L}^2 | (LS)JM \rangle = L(L+1) = J(J+1) + S(S+1) - 2 \hat{d}_S J(J+1)
\]

\[
\hat{\alpha}_S = \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}
\]

\[
\alpha_L = 1 - \alpha_S = 1 - \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)}
\]

\[
g_j = g_S = 2\alpha_S = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}
\]

**Primer:** Elektron v stanju z orbitalno vrtilno količino \( L \) s spinom \( S = \frac{1}{2} \).

\[
g_{1/2} = \frac{1}{6} \quad g_{3/2} = \frac{4}{3}
\]

\[
\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}
\]

\[
\mu_B B \frac{4M}{3}
\]

\[
\mu_B B \frac{M}{3}
\]
IZRAZIMO MATRIČNI ELEMENT COULOMBSKE INTERAKCIJE MEDI DVEMA ELEKTRONOIMA

\[
\frac{1}{r_{12}} = \sum_{\ell_2} \frac{r_{12}^{\ell_2}}{r_{12}^{\ell_2+1}} \hat{C}^{(1)} + \hat{C}^{(2)} = \sum_{\ell} \frac{(-1)^{\ell}}{\sqrt{2\ell + 1}} \frac{r_{12}^{\ell}}{r_{12}^{\ell+1}} \left\{ \hat{C}^{(1)} \hat{C}^{(2)} \right\}_0
\]

Spada u množico operatorjev oblike

\[
\hat{G}_{12} = \gamma(r_{12}) \left\{ \hat{t}^{\alpha_1}(1) \hat{t}^{\beta_2}(2) \right\}_Q \hat{a}_1 \hat{a}_2 \text{ neodvisna od spin }
\]

IZRAZIMO MATRIČNI ELEMENT

\[
\langle ab | \gamma(r_{12}) \left\{ \hat{t}^{\alpha_1}(1) \hat{t}^{\beta_2}(2) \right\}_Q | cd \rangle
\]

Med nesklapljivimi eno elektronskimi stanji (orbitalami) \(|a> = |m_a m_a^a m_e^a>, m_e^a\rangle\)

\[
\langle m_e m_e, m_e | \frac{1}{r} P_{m_e}(r) Y_{m_e}^{\ell}(\theta, \phi) X_{m_s} \rangle = \text{spinika funkcija s projekcijo } m_s
\]

\[
\langle ab | \gamma(r_{12}) \left\{ \hat{t}^{\alpha_1}(1) \hat{t}^{\beta_2}(2) \right\}_Q | cd \rangle = \int P_a(r_1) P_b(r_2) \gamma(r_{12}) P_c(r_1) P_d(r_2) dr_1 dr_2 \sum_{\lambda_1, \lambda_2} \langle a | \hat{t}^{\lambda_1}(1) \hat{t}^{\lambda_2}(2) | d \rangle \langle b | \hat{t}^{\lambda_1}(1) \hat{t}^{\lambda_2}(2) | c \rangle \times \Delta(m_s^a, m_s^c) \Delta(m_s^b, m_s^d)
\]

Integrал čet kotne koordinate 1

\[
(-1)^{q_1 - q_2 + \ell} (q_1 q_2 - K) \sqrt{2K + 1}
\]

Integrál čet kotne koordinate 2
W.E. \[ <a l_i^{\pm} c^{\pm}_{\pm} | c> = (-1)^{l_a-l_b} (l_a \pm l_c) \langle a l_i l_{\pm}^{\pm} l_{\pm}^{\pm} c | c> \]

\[ <b l_i^{\pm} c^{\pm}_{\pm} | c> = (-1)^{l_b-l_c} (l_b \pm l_d) \langle b l_i l_{\pm}^{\pm} l_{\pm}^{\pm} c | c> \]

\[ \rightarrow <a b | \hat{\sigma} (r_{1}, r_{2}) \exp \{ \pm i \hat{r}_{2} \hat{r}_{2} \} | c d> = \int \frac{P_{a}(r_{1}) P_{b}(r_{2}) \gamma (r_{1}, r_{2}) P_{c}(r_{1}) P_{d}(r_{2}) dr_{1} dr_{2}}{(-1)^{l_a + l_b + l_{\pm}^{\pm} - l_{\pm}^{\pm} + Q - l_{\pm}^{\pm} - l_{\pm}^{\pm} \sqrt{2k+1}} \langle a l_i | c d> \langle b l_i | c d> S_{a}^{c} S_{b}^{d} \]

\[ \sum_{l_{\pm}^{\pm}} (l_{a} \pm l_{c}) (l_{b} \pm l_{d}) (a b l_{\pm}^{\pm} c) \]

Coulomb's Integral:

\[ <a b | \frac{1}{r_{12}} | c d> = \sum_{k=0}^{\infty} \left( \int \frac{P_{a}(r_{1}) P_{b}(r_{2}) r_{12}^{k}}{r_{12}^{k+1}} P_{c}(r_{1}) P_{d}(r_{2}) dr_{1} dr_{2} \right) (-1)^{l_{a}+l_{b}-l_{c}^{\pm}-l_{d}^{\pm}} \]

\[ \langle a l_i l_{\pm}^{\pm} l_{\pm}^{\pm} c | c d> \]

\[ \sum_{l} (l_{a} \pm l_{c}) (l_{b} \pm l_{d}) (a b l_{\pm}^{\pm} c) \]

Slater's integral \( R^{2}_{a b}(a b, c d) \)
REDUCIRANI MATRIČNI ELEMENT TENZORSA "C":

\[ <l m | C^a_{\ell m} | l' m'> = \sqrt{\frac{4\pi}{2\ell+1}} \int Y^a_{\ell m}(\Omega) Y^{\ast}_{\ell' m'}(\Omega) Y^\ast_{\ell m}(\Omega) d\Omega = (-1)^{l-m} \left( \frac{\ell \times \ell'}{2m} \right) (0 0 0) \]
NADALJUJEM Z MATRIZNIH ELEMENTO:

\[ \langle ab | \frac{A}{m_2} | cd \rangle = \sum_{k2} \left[ R^2_{k2}(ab,cd) \sqrt{\frac{(2k_a+1)(2k_b+1)(2k_c+1)(2k_d+1)}{2k+1}} \right] (-1)^k (l_a \leq l_c) (l_b \leq l_d) (o_0 0 0 0) (-1)^{m_a + m_b} \sum (m_s^a, m_s^c) (m_s^b, m_s^d) (-1)^2 (l_a \leq l_c) (l_b \leq l_d) (-1)^2 (m_a \leq m_c) (-m_b \leq m_d) ] \]

SKLAPAJEME 3 VRTLNIH KOLICIN